THEORETICAL STUDY OF HARMONIC GENERATION SOUND BEAMS IN CASE OF UNIFORM, EXPONENTIAL AND COSINUSOIDAL APERTURES

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Abstract- **The non-linear propagation of ultrasound in medical imaging has recently been exploited to improve image resolution and remove near field artifacts generated by overlying tissue structures. The images are formed using the second harmonic energy generated by nonlinear propagation. Second harmonic beams have narrower beam width and lower side lobes than the fundamental. The second harmonic draws energy from the fundamental continuously along the propagation path. These characteristics contribute to improve the quality of medical ultrasound images.**

In this paper, our objective is to show that the choice source aperture has significant consequences on the quality of the field transmitted in term of directivity. We also studied the field generated by three apertures sources, in a nonlinear medium. Theoretical approach, based on the Khokhlov-Zabolotskaya-Kuznetsov (KZK) parabolic approximation is used in order to consider the diffraction effects related to the use of a focusing real source. The fundamental ultrasonic fields and the second harmonic are compared at three distances from the source. Width and side lobes were used when recording three distributions of the beam form.

***Keywords* – Nonlinear, ultrasound, harmonic, KZK**

I. INTRODUCTION

It has been shown since that several physical phenomena related to ultrasound propagation can not be explained by linear theory [1]. Among these phenomena one finds the nolinearity, the diffraction and the absorption. Indeed the local variations of velocity and density can not be neglected. Thus it is necessary to enhance these approximations and then obtain a non linear differential equations system. These variations result in a deformation of the temporal profile of the emitted ultrasonic wave. This distortion manifests itself in frequency domain as the harmonics generation with multiple components of fundamental frequencies which did not exist in the spectrum of the emitted signal. This phenomenon is more significant when the intensity or the frequency of the wave are high [2].

Linearity has an effect on all the methods of measurement and characterization of the mediums and ultrasounds equipment. For this reason, it is primordial to consider that in the definition, design and the development of the equipment using the ultrasonic waves. A theoretical study is essential as a preliminary to demonstrate the distortion of the wave along the propagation in order to be able to simulate the experimental study. The general wave equation which is valid for no linearity, diffraction, and absorption in sound beams was derived by Kuznetsov [3] Zabolotskaya and Khokhlov [4]. This model was obtained starting from the physical basic laws by improving the approximations of the second order linear theory. It was validated by several researchers [5, 6].

The objective of this work is the theoretical study of diffraction effects in the case of real sources [7, 8]. We present, here particularly, the case of a focusing circular source. Three apertures: uniform, cosine and exponential were studied. We analysed the case of an emission at 3 MHz in a medium whose acoustic properties are similar to water. A calculation algorithm, based on parabolic approximation KZK, was carried out using the finite differences method.

This theoretical study makes it possible to determine the harmonics generation in the propagation medium. We are interested particularly by the levels of the side lobes and directivity of the fundamental main beam frequency and to the second harmonic. The obtained results show that we can obtain a more directing beam with narrower beam width and lower side lobes by the use of an exponential aperture. The choice of an adapted space distribution at the plan source improves the lateral resolution and the images contrast formed with the second harmonic frequency [9, 10].

II. MODEL EQUATION

For the acoustic non linearity study, we based our analysis on the KZK parabolic equation. This model takes into account the diffraction, absorption and distortion. It supposes that the distortion is negligible at distances lower than the wavelength and that it is valid only in the case of the ultrasonic progressive and narrow beams. In the case of a focusing source, the non-linear parabolic wave equation is [11]:

$$\frac{\partial^2 P}{\partial\sigma\partial\tau} - \frac{1}{4G}\nabla_\perp^2 P - A\frac{\partial^3 P}{\partial\tau^3} = \frac{B}{2}\frac{\partial^2 P^2}{\partial\tau^2} \quad (1)$$

where $\sigma = z/d$ is a dimensionless range in term of axial coordinate and the focal length d, $\tau = \omega(t - z/c_0)$ dimensionless retarded time where $\omega = 2\pi f$ with f the source frequency, c_0 the source sound speed and $P=p/p_0$ is a dimensionless pressure in terms of the acoustic pressure p and its on-source pressure p_0. The two dimensional Laplace operator is applied with respect to the dimensionless vector

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$\xi=r/a$, where $r=(x,y)$ is the transverse coordinate vector and a is the source radius. For an axisymmetric sound field we may write:

$$\nabla_{\perp}^{2}=\partial^{2}/\partial\xi^{2}+\xi^{-1}(\partial/\partial\xi).$$

The focal plane is defined to be at $\sigma=1$, with the source located at $\sigma=0$ and radiating in the $+\sigma$ direction.

The characteristic of the sound beam is given by the dimensionless parameters :

$A=\alpha d$ with $\alpha=\delta\omega^{2}/2c_{0}^{3}$ (thermo viscous attenuation) and δ the sound diffusivity,

$B=d/l_{d}$, source amplitude, with $l_{d}=\rho_{0}c_{0}^{3}/\beta\omega p_{0}$ the plane wave shock formation distance, where ρ_{0} is the density of the medium and β the coefficient of no linearity.

$G=z_{0}/d$ is the linear focusing gain, where $z_{0}=\omega a^{2}/2c_{0}$ is the Rayleigh distance of the unfocused source.

Let us now consider the case of a source which is periodic in time, with the period $2\pi/\omega$. We search a solution of equation (1) in form of a Fourier series:

$$P=\sum_{n=1}^{+\infty}(g_{n}\sin n\tau+h_{n}\cos n\tau)=\sum_{n=1}^{+\infty}P_{n} \qquad (2)$$

where g_{n} and h_{n} are functions of the spatial coordinates. By replacing P by this value in the previous equation (1), we then obtain a set of coupled partial equations for g_{n} and h_{n} [11].

$$\frac{\partial g_{n}}{\partial\sigma}=-n^{2}Ag_{n}+\frac{1}{4Gn}\nabla_{\perp}^{2}h_{n} +n\frac{B}{2}\Big(\frac{1}{2}\sum_{p}^{n-1}(g_{p}g_{n-p}-h_{p}h_{n-p})-\sum_{p=n+1}^{N}(g_{p-n}g_{p}+h_{p-n}h_{p})\Big)$$

$$\frac{\partial h_{n}}{\partial\sigma}=-n^{2}Ah_{n}+\frac{1}{4Gn}\nabla_{\perp}^{2}g_{n} +n\frac{B}{2}\Big(\frac{1}{2}\sum_{p}^{n-1}(h_{p}g_{n-p}+g_{p}h_{n-p})+\sum_{p=n+1}^{N}(h_{p-n}g_{p}-g_{p-n}h_{p})\Big)$$

We based our argument on this partial equation to study the radiation diagram of the fundamental and the second harmonic.

The algorithm calculation based on modified Bergen code is initialised by one of the three apertures (figure 2).

The parameters for the theoretical model are :

Source radius $a = 1$ cm, initial pressure $P_{0}= 1{,}2$ bar, frequency $f_{0} = 3$ MHz

$\alpha = 0.064$ Np/m , c_{0}=1500 m/s, $\rho_{0}= 10^{3}$ kg/m^{3}, $\beta = 3.5$

$d = 10$ cm, $\Delta\sigma = 25\ 10^{-4}$, $\Delta\xi=10^{-2}$, $\xi_{max}= 5$, $\sigma_{max}= 50$, harmonic number $N=12$.

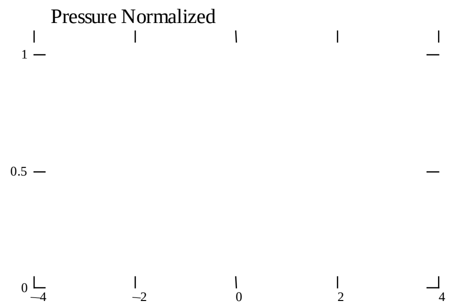
III. Initial aperture

We will study the diagrams of radiation associated with the three following source conditions : (1) exponential aperture, (2) cosine aperture and (3) uniform aperture (figure1). We considered the pressure amplitude in the source plan.

(1)- $g_{1} = \exp(-4\xi^{2})$, $\xi < 1$; 0 otherwise
$g_{n>1} = h_{n} = 0$

(2)- $g_{1} = \cos(\pi\xi/2)$, $\xi < 1$; 0 otherwise
$g_{n>1} = h_{n} = 0$

(3)- $g_{1} = 1$, $\xi < 1$; 0 otherwise
$g_{n>1} = h_{n} = 0$



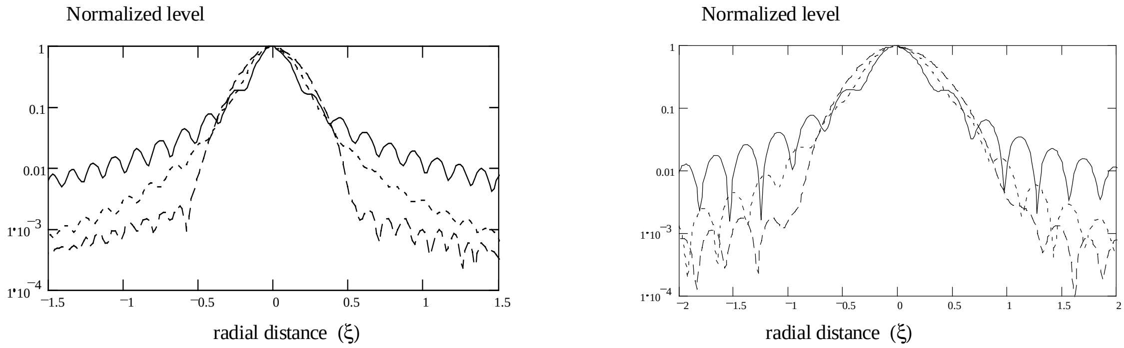


Fig. 1. Numerical results for beam patterns for the fundamental components in water(—uniform, …. cosine, ---exponential)

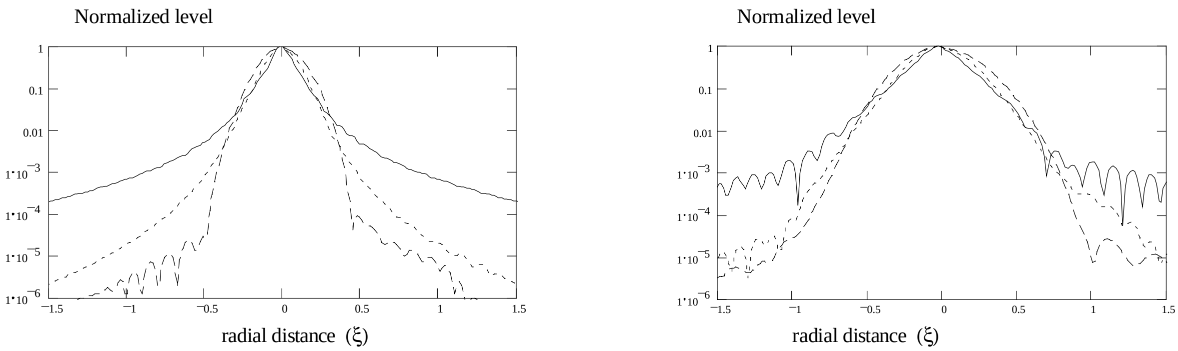


Fig. 2. Numerical results for beam patterns for the second harmonic components in water (—uniform, …. cosine, ---exponential)

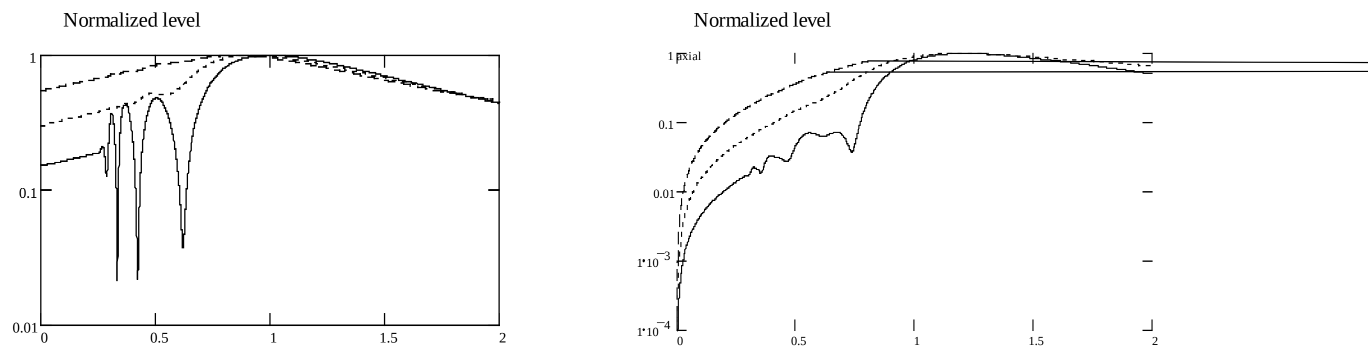


TABLE 1

BEAM WIDTH AT (-40 dB) OF THE FUNDAMENTAL IN mm

	focal point	farfield
Exponential aperture	34	47
cosine aperture	42	57
uniform aperture	84	95

TABLE 2

BEAM WIDTH AT (-80 dB) OF THE SECOND HARMONIC IN mm

	focal point	far field
exponential aperture	30	55
cosine aperture	35	58
uniforme aperture	61	78

B. Axial distributions

In figure 3, the axial distributions are given for the fundamental and second harmonic on for that three apertures. We can notes, for distribution 3, the presence of oscillations in the near field. They are due to the variations of the phase in this range. This range is limited by the last maximum of the fundamental signal which corresponds to the focal distance. After this focal point the magnitudes of the field decrease. The second harmonic fields for the three apertures come up to a maximum at a distance of 12,5 cm and after this the signal decreases smoothly. This decrease is more significant in the case of aperture 3.

V. CONCLUSION

In this study we compared three distributions of ultrasonic pressure in the source plan. The principal idea is to see the characteristics of the fundamental and the second harmonic generated by a focusing circular source. This analysis permits us to predict the distortion of the wave according to the distribution of the emitted beam and the parameters of the propagation medium.

Indeed, the quality of the images obtained by ultrasounds is closely related to the used frequency and the nature of the emitted field. The choice of a suitable excitation makes it possible to minimize in a significant way the amplitude of the side lobes and the width of the main lobe and consequently to improve the radiation pattern of the transducer. We can according to the theoretical results obtained that the distribution of the second harmonic shows several interesting characteristics for the imaging. Among these characteristics we have, a more directing beam , a focal spot of reduced size and a very attenuated side lobes.

The fundamental and the second harmonic beams, generated by an exponential excitation are more directing than that obtained with the two others. The second harmonic remains very directing with low side lobes. This theoretical work enabled us to show that the choice of the excitation in the plan source has significant consequences on quality of the field transmitted and by consequence on the quality of a ultrasounds imaging system.

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